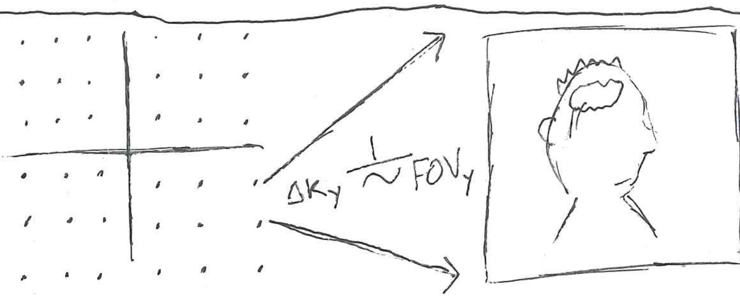
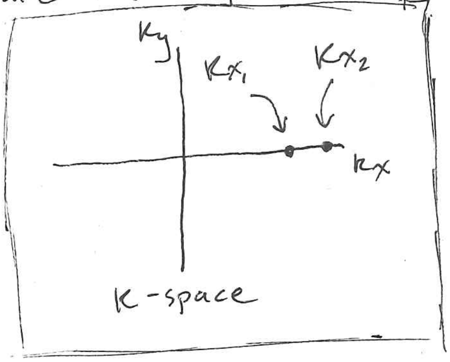
WHY DOES SAMPLING INTERVAL IN K-SPACE DETERMINE FIELD OF VIEW?

MUMMY 2017



CONSIDER THE FOLLOWING:

Take two adjacent samples in k-space, Kx_1 and Kx_2, both on the Kx axis:



Since they are adjacent, their corresponding spatial frequencies can be written

$$\boxed{\begin{aligned} f_1 &= m \cdot \Delta K_x \\ f_2 &= (m+1)\,\Delta K_x \end{aligned}}$$

for some integer m.

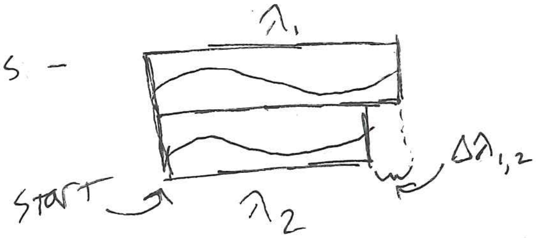
Thus, their wavelengths are

$$\boxed{\begin{aligned} \lambda_1 &= \frac{1}{m \cdot \Delta K_x} \\ \lambda_2 &= \frac{1}{(m+1)\Delta K_x} \end{aligned}}$$

THINK ABOUT STARTING at some point in image space and traveling in the positive x direction. As you do, you are moving "along" the waves corresponding to Kx_1 and Kx_2.

With every cycle of wave Kx_1, the slightly shorter wave Kx_2 falls slightly behind, specifically a distance $\Delta\lambda_{1,2} = \lambda_1 - \lambda_2 = \frac{1}{m\Delta K_x} - \frac{1}{(m+1)\Delta K_x} = \frac{1}{m(m+1)\Delta K_x}$.

That is -



and so on.

So after m cycles of wave Kx_1, we have traveled a distance $m \cdot \lambda_1 = m \cdot \frac{1}{m\Delta K_x} = \frac{1}{\Delta K_x}$ AND wave Kx_2 has fallen "behind" a distance

$$m\,\Delta\lambda_{1,2} = \frac{m}{m(m+1)\Delta K_x} = \frac{1}{(m+1)\Delta K_x} = \lambda_2 .$$

CONCLUSION: at this point a distance $\frac{1}{\Delta K_x}$ from where we started, (and no sooner!) both waves are simultaneously at integer multiples of their respective wavelengths relative to where we started. Everything has "started over".

In this way does the k-space sampling interval determine FOV.